Global Defects In Theories With Lorentz Symmetry Violation.

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Abstract

We study global topological defects in the Jacobson-Corley model which breaks Lorentz symmetry and involves up to fourth order derivatives. There is a window in the parameter space in which no solution exists. Otherwise, different profiles are allowed for the same values of the parameters. For a scale of Lorentz violation much higher than the scale of gauge symmetry breaking, the energy densities are higher, of the same order or smaller than in the usual case for domain walls, cosmic strings and hedgehogs respectively. Possible cosmological implications are suggested.

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1 Introduction.

The possibility that Lorentz symmetry may be violated in some ways has been extensively analyzed in recent years. In particular, the robustness of the predictions of inflation with respect to possible transplanckian effects, has been analyzed in this context as well as the Hawking radiation of black holes, the dark energy scenarios and the particle reactions at LHC [1, 2, 3]. Recently, the possibility that Lorentz symmetry may be violated has been analyzed in quantum electrodynamics [4]. It may help to evade the GZK cut off of the ultra high energy cosmic rays [5, 6]. Its possible impact on neutrino oscillation has also attracted attention [7, 8].

It has been argued and demonstrated in some cases that quantum gravity leads to effective theories with Lorentz symmetry violation, like non commutative field theory. Some models also violate Lorentz symmetry, but by changing the dispersion relations in a way which keeps locality [9, 10, 11, 12]. These models break boosts but keep translation and rotation symmetry untouched. This is achieved by modifying the kinetic term, introducing a supplementary term which involves the three dimensional Laplacian. This extra term can be rewritten in a manifestly covariant way by introducing a unit time-like vector which is also dynamical. Let us point the fact that theories like k-inflation or k-essence have modified kinetic terms although Lorentz symmetry is kept valid.

For most models, particle physics interactions have been studied, especially the dependence of the thresholds on the new scale. One knows that in the standard model and the grand unification theories, the sector of classical solutions is important. Topological defects have been analyzed in contexts where Lorentz symmetry is violated like non commutative theories [13, 14, 15, 16, 17, 18, 19].

In this letter we briefly analyze how solitons are affected by a breakdown of the Lorentz symmetry but within a theory which is still local. For simplicity we focus on global defects and restrict ourselves to the Jacobson-Corley model. This work is organized as follows. The second section is devoted to a brief survey of the status of solitons in field theory on a non commutative space; it will serve as a basis for comparison. In the third section we briefly introduce the Jacobson-Corley dispersion relation and the Lagrangian associated to it. Adding a Higgs potential, we work out the field equation for a domain wall and show that one has more than one profile interpolating between the two vacuua. The fourth and fifth sections repeat the same

analysis for cosmic strings and hedgehogs. We study the energy densities of these configurations and extrapolate our results to draw potential cosmological implications.

2 Solitons in non commutative theories

Let us consider the non commutative field theory in $2+1$ dimensions defined by the commutation relation

$$[x, y] = i\theta \quad . \tag{1}$$

The energy functional is the two dimensional integral

$$E = \frac{1}{g^2} \int d^2 z (\partial_z \phi \star \partial_{\bar{z}} \phi + \theta V(\phi)) \quad , \tag{2}$$

where the star product, in terms of a rescaled complex variable, reads

$$(A \star B)(z, \bar{z}) = \left[\exp\left(\frac{1}{2} (\partial_z \partial_{\bar{z}'} - \partial_{z'} \partial_{\bar{z}}) \right) A(z, \bar{z}) B(z', \bar{z'}) \right]_{z'=z} \quad . \tag{3}$$

The complex variable z parameterizes the complex plane: $z = x + iy$.

When studying solutions with rotational symmetry and finite energy, it is simpler to consider big values of θ; one can then neglect the kinetic term. The field equations read, in this limit,

$$\frac{\partial V}{\partial \phi} = m^2 \phi + b_3 \phi \star \phi + b_4 \phi \star \phi \star \phi = 0 \tag{4}$$

In the commutative case the only the configurations satisfying this equation are given by $\phi = \lambda_i$ where the λ_i are minima of V. However, when non commutativity sets in, one obtains an infinite number of solutions:

$$\phi = \sum c_n \phi_n(r^2) \quad , \quad \phi_n(r^2) = 2(-1)^n e^{-r^2} L_n(r^2) \quad . \tag{5}$$

The L_n are Laguerre polynomials while the c_n are real numbers chosen among the extrema λ_i of the potential. The energy densities of these solutions are proportional to the scale of non commutativity:

$$E \sim \frac{2\pi\theta}{g^2} \sum_{n=0}^{\infty} V(c_n) \tag{6}$$

Some important remarks can be drawn from these formulas. First, the number of solutions is infinite in this model; this is linked to the fact that the Lagrangian of this model contains an infinite number of derivatives. The second one is that solutions exist even in the limiting, unphysical case where the non commutative scale is high. Finally, the energy density is proportional to that scale. We will basically analyze how these characteristics are present or not in the Jacobson-Corley model.

3 Domain walls.

The Jacobson-Corley dispersion relation was initially used to study possible transplanckian imprints on the Hawking radiation. It reads

$$\omega^2 = k^2 + \mu k^4 \quad . \tag{7}$$

The parameter μ sets the scale where the violation of Lorentz symmetry sets in. As we wish to avoid a cut-off on momenta(or avoid imaginary frequencies), we will take μ to be positive. At small momenta, one has the usual mass-energy relation.

The Lagrangian which leads to domain wall solutions is constructed with a real scalar field and possesses Z_2 symmetry. Putting together the kinetic term which leads to the Jacobson-Corley dispersion relation with a Higgs potential, we have

$$\mathcal{L} = \frac{1}{2}\eta^{\rho\tau}\partial_\rho\phi\partial_\tau\phi - \frac{\mu}{2}(\Delta\phi)^2 - \frac{\lambda}{4}(\phi^2 - v^2)^2 \tag{8}$$

The three dimensional Laplacian acting on the field $\Delta\phi$ breaks the symmetry under boosts; it can be rewritten as a four dimensional operator by introducing a unit vector [20].

We shall use the dimensionless length variable x defined by $x = z/\sqrt{\mu}$. The Higgs field will be parameterized as $\phi = vf(x)$. Its dependence on the space coordinate is dictated by the following differential equation ($f^{(n)}(x)$ is the derivative of order n of the function $f(x)$):

$$f^{(4)}(x) - f^{(2)}(x) + \alpha f(x)(f^2(x) - 1) = 0 \quad \text{where} \quad \alpha = \lambda\mu v^2 \tag{9}$$

is the dimensionless parameter giving the square of the ratio of the two masses: the one linked to the gauge scale by the one related to the Lorentz

breaking scale. We will assume that the violation of Lorentz symmetry takes place at a very high energy scale, much higher than the vacuum expectation value of the Higgs field. Thus, the parameter α will be tiny.

The boundary conditions are $f(-\infty) = -1$ and $f(\infty) = 1$. Lets us analyze the behavior of the field in the asymptotic region. This is done by writing, in the region $x \to \infty$, the decomposition $f(x) = 1 + g(x)$. The function $g(x)$ has to vanish in this limit. It obeys the differential equation

$$g^{(4)}(x) - g^{(2)}(x) + 2\alpha g(x) = 0 \tag{10}$$

which solution can be written as

$$\begin{aligned} g(x) &= \sum_{k=1}^{4} C_k \exp\left(\beta_k x\right) \quad \text{with} \\ \beta_1 &= -\beta_2 = \frac{1}{\sqrt{2}}\sqrt{1-\sqrt{1-8\alpha}} \quad , \\ \beta_3 &= -\beta_4 = \frac{1}{\sqrt{2}}\sqrt{1+\sqrt{1-8\alpha}} \quad . \end{aligned} \tag{11}$$

As explained above we are interested in small values of α. The general solution can therefore be written as

$$\begin{aligned} g(x) &= C_1 \exp\left(\sqrt{2\alpha}\, x\right) + C_2 \exp\left(-\sqrt{2\alpha}\, x\right) + C_3 \exp\left((1-\alpha)x\right) \\ &+ C_4 \exp\left(-(1-\alpha)x\right) \end{aligned} \tag{12}$$

so that one has two normalizable solutions, corresponding to $C_1 = C_3 = 0$. At the other spatial infinity($x \to -\infty$), writing $f(x) = -1 + g(x)$, one ends up with the same differential equation displayed in Eq.(10). Now, the normalizable modes correspond to $C_2 = C_4 = 0$ in Eq.(12).

One sees that contrary to the orthodox model which leads to a second order differential equation, one will have here more than one configuration obeying the same boundary conditions:

$$\begin{aligned} f(x) &\to -1 + C_1 \exp\left(\sqrt{2\alpha}\, x\right) \text{as}\, x \to -\infty \,\text{and} \\ f(x) &\to 1 + C_2 \exp\left(-\sqrt{2\alpha}\, x\right) \text{as}\, x \to \infty \end{aligned} \tag{13}$$

is one of them

$$\begin{aligned} f(x) &\to -1 + C_3 \exp\left((1-\alpha)x\right) \text{as}\, x \to -\infty \,\text{and} \\ f(x) &\to 1 + C_4 \exp\left(-(1-\alpha)x\right) \text{as}\, x \to \infty \end{aligned} \tag{14}$$

is another. The symmetry under parity imposes the equalities $C_2 = -C_1$ and $C_4 = -C_3$. In the first solution, the field goes to its asymptotic behavior much slower than the second one; it has a bigger spread. The fact that the two configurations have vanishing Higgs fields at the origin implies that the one which goes to the boundary value slower has a sharper slope at the origin; the interval on which one has to integrate is smaller. Essentially, these two behaviors conspire to give comparable total energies to the two configurations.

The density of energy per unit length is

$$\sigma = \int dz T_0^0 \sim \kappa(\alpha) \frac{v^2}{\sqrt{\mu}} \quad , \tag{15}$$

in contrast to the case where Lorentz symmetry is present for which the formula

$$\sigma \sim \sqrt{\lambda} v^3 \tag{16}$$

holds. The constant $\kappa(\alpha)$ also depends on the behavior at infinity which is chosen. If, as it is customary, one takes the scale at which Lorentz violation takes place to be the Planck one, and the vacuum expectation value of the Higgs field to be at the GUT scale(10^{16} GeV), one obtains essentially that $\kappa(\alpha) \sim 0.1$ so that the domain wall we obtain here are two orders of magnitude heavier than in the unmodified theory. Since domains walls dominate the energy density of the universe after a time corresponding to

$$t_d = \frac{1}{8\pi G \sigma} \quad , \tag{17}$$

their take over will be much quicker in our model, in the absence of inflation.

Let us finally remark that if the scale at which the Lorentz violation takes place is close enough to the scale of gauge symmetry breaking($\alpha \geq 1/8$), the quantities β_k which control the behavior of the field at infinity become complex. For example, for $\alpha = 1$ one has $\beta_1 = 0.813442 - 0.813135i$ and $\beta_2 = 0.978318 + 0.676097i$. As no combination of the solutions is real, this means there is no domain wall solution. This is pretty different for the orthodox theory. It means that in the Corley-Jacobson model we are analyzing, if the scale of gauge symmetry breaking is sufficiently close to the one of Lorentz violation, no domain wall solution exists. This is a safer situation in the sense that these objects are a problem from the cosmological point of view;

inflation is not necessary in this setting in order to get rid of them. However, inflation will still be needed to generate the initial density inhomogeneities.

Let us finally emphasize another important point related to the sign of the parameter μ. The energy of the configuration reads

$$E = \int dz \left[\frac{1}{2}(\partial_z \phi)^2 + \frac{\mu}{2}(\partial_z^2 \phi)^2 + \frac{\lambda}{4}(\phi^2 - v^2)^2 \right] \quad . \tag{18}$$

For a positive μ, this integrand is a sum of three positive quantities; the integral converges only if all of them go to zero at spatial infinity; this means as usual that the field must be in the minimum of the potential in that region. On the contrary, when μ is negative, the second term has a negative sign, contrary to the two others, so that it becomes possible for the integrand to vanish in the far region without the field being in the minimum of the potential there. The obvious choice $\phi(\pm\infty) = \pm v$ is allowed but it is not the only one.

Let us now come to the numerical treatment. In the orthodox theory, the equation of the domain wall profile is of second order. This means that two initial conditions are necessary to fix unambiguously a solution. The vanishing of the field at the origin(due to symmetry arguments) leaves the first derivative as a parameter which is then chosen(by the shooting method for example) to satisfy the boundary conditions. In our case, the equation is of fourth order. The value of the field on the wall is fixed like in the usual case; one then chooses the first and second derivatives and fixes the third one to attain the prescribed value at infinity. For example, taking $\alpha = 0.1$, and $f^{'}(0) = 0.233806, f^{(2)}(0) = 0, f^{(3)}(0) = -0.0214249$, one finds a solution which attains its asymptotic value at $x = 10$.

4 Cosmic Strings.

Let us now turn to the topological defect having one more dimension, i.e the cosmic string. The global string will be a solution of the system driven by the Lagrangian

$$\mathcal{L} = \eta^{\rho\tau}(\partial_\rho \Phi)(\partial_\tau \Phi)^+ + \mu(\Delta\Phi)(\Delta\Phi)^+ - \frac{1}{2}\lambda(\Phi\Phi^+ - v^2)^2 \tag{19}$$

which displays a $U(1)$ symmetry. The Ansatz has the form

$$\Phi = v f(r/\sqrt{\mu}) \exp{(i\theta)} \quad . \tag{20}$$

Using the dimensionless length x and the ratio between the scales defined as α in the preceding section, the differential equation to be solved is

$$\begin{aligned} f^{(4)}(x) &+ \frac{2}{x} f^{(3)}(x) - \left(1 + \frac{3}{x^2}\right) f^{(2)}(x) + \left(\frac{3}{x^3} - \frac{1}{x}\right) f^{'}(x) + \alpha f^3(x) \\ &+ \left(-\frac{3}{x^4} + \frac{1}{x^2} - \alpha\right) f(x) = 0 \quad . \end{aligned} \tag{21}$$

Let us now see what happens at the boundaries. At the origin, the fields must vanish in order for the configuration to be regular. Writing $f(x) \sim x^t$, one obtains the value

$$t = 3 \quad . \tag{22}$$

At infinity one has to be in the vacuum so that the energy density vanishes there. The counterpart of Eq.(10) reads

$$2\alpha g(x) - \frac{g^{'}(x)}{x} - g^{''}(x) + 2\frac{g^{(3)}(x)}{x} + g^{(4)}(x) = 0 \quad . \tag{23}$$

and its most general non singular solution is

$$g(x) = C_2 \exp\left(\beta_2 x\right) + C_4 \exp\left(\beta_4 x\right) \tag{24}$$

where β_2, β_4 have been given in Eq.(11); the terms we neglected are effectively small. So, in principle one can build different solutions; they begin in similar ways near the origin but go to their value at infinity with different rates.

Concerning the energy density, one has now to integrate on the plane orthogonal to the string. This gives an extra factor $\sqrt{\mu}$ so that the energy density per unit length reads

$$E = 2\pi v^2 \kappa(\alpha) \tag{25}$$

where $\kappa(\alpha)$ is a dimensionless function. We have put a cut-off at a radius where the field attains 95 percent of its asymptotic value. If gauge symmetry is broken at the GUT scale while Lorentz symmetry breaking takes place at the Planck scale, this number is of order of a few hundreds. Basically, the difference with the usual case is not as important as for domain walls. This means such defects will behave as the ones of the orthodox theory.

5 Hedgehogs.

The Lagrangian

$$\mathcal{L} = \eta^{\rho\tau}(\partial_\rho \Phi^a)(\partial_\tau \Phi_a) + \mu(\Delta\Phi^a)(\Delta\Phi_a) - \frac{1}{2}\lambda(\Phi^a\Phi_a - v^2)^2 \quad , \tag{26}$$

where the sum on the index a goes from 1 to 3 is $SO(3)$ symmetric. The Ansatz for the global monopole is, in spherical coordinates,

$$\begin{aligned} \Phi_1 &= v f(r/\sqrt{\mu}) \sin\theta \cos\phi \quad , \quad \Phi_2 = v f(r/\sqrt{\mu}) \sin\theta \sin\phi \quad , \\ \Phi_3 &= v f(r/\sqrt{\mu}) \cos\theta \quad . \end{aligned} \tag{27}$$

The profile function obeys the differential equation

$$f^{(4)}(x) + \frac{4}{x} f^{(3)}(x) - \left(1 + \frac{4}{x^2}\right) f^{(2)}(x) - \frac{2}{x} f'(x) + \left(\frac{2}{x^2} - \alpha\right) f(x) + \alpha f^3(x) = 0 \quad . \tag{28}$$

In the asymptotic region, one has

$$f(x) = 1 + \frac{1}{x}\left(C_2 \exp(\beta_2 x) + C_4 \exp(\beta_4 x)\right) \quad . \tag{29}$$

Note the difference with the previous defects embodied by the extra $1/x$ factor. Near the origin one has $f(x) \sim x^3$, just as for the cosmic string.

Now, the energy density has to be integrated over the three dimensional space; this results in the formula

$$E = 4\pi v^2 \sqrt{\mu}\kappa(\alpha) \quad ; \tag{30}$$

from this one sees that monopoles are much lighter in this model.

Extrapolating this, it is likely that local monopoles will be much lighter in this model; on dimensional grounds one can argue that introducing a gauge field brings in a coupling constant. This will be verified provided that the dependence of the energy on this constant is mild, as in the usual case.

6 Conclusion.

We have analyzed topological defects in the Jacobson-Corley model. Like in the non commutative case, the solutions are not unique. However, they

display a different dependence on the scale at which the Lorentz symmetry is broken. The hedgehogs are the only ones which energy densities are proportional to that scale, like in non commutative theories. On the other hand, domain walls are much heavier. As monopoles are lighter in this model, they will display a smaller deficit angle. The situation for cosmic strings will be roughly like in the usual case.

This work can be extended in two ways. First, one may write down the full equations for local defects. This would lead to configurations with a perfectly integrable energy density. However, the equations are much more involved while the main characteristics are likely to be the same. A second point is the study of finite temperature field theory, which would reveal the details of the formation of these defects. In the parameter space where no defect exists, it is important to know if there is a restoration of gauge symmetry. If this was the case, the Kibble mechanism would take place and the question would then be to know how the formed configurations disappear as the universe cools.

Let us finally point out that in k inflation, the kinetic term is also of an order higher than two, but without symmetry violation. A treatment similar to our may be of interest in that setting.

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